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PARTIALLY BALANCED INCOMPLETE BLOCK DESIGNS WITH THREE REPLICATES

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SUMMARY

A series of partially balanced incomplete block (PBIB) designs with three replicates has been constructed. Method of analysing the data obtained by adopting such a design is also indicated.

Keywords : Partially balanced, dual designs, triplet, initial block, pseudo variance-covariance matrix.

Introduction

Block designs are extensively used in many fields of research. Although a large number of balanced and partially balanced incomplete block (PBIB) designs are available in literature, it may not be possible to get a suitable design in particular practical situations.

If the number of treatments to be compared is large, it is desirable for experimenter for reasons of economy or dearth of experimental material to have a small number of replications (two or three). Ramakrishna [3] and Mohan [2] gave 5-associate PBIB designs with two replicates by dualizing suitably chosen group divisible (G. D.) designs. For a design with paramaters $v = 2a^2$, $b = 3a$, $r = 3$ and $k = 2a$ either a BIB design or PBIB design with two associate classes is not available except for $a = 2$. Therefore, for the above parameters the most efficient design is a 3-associate PBIB design, if such a design exists. In the present investigation, construction and analysis of 3-associate PBIB designs with three replicates for the above mentioned parameters are described.

2 Construction

Let us take $3a$ ($a \geqslant 3$) symbols which are arranged and classified into three groups as follows :

I	group	1	2	3	...	a
II	group	$a+1$	$a+2$	$a+3$	...	$2a$
III	group	$2a+1$	$2a+2$	$2a+3$	...	$3a$

Form all possible a^2 pairs of the type (X, Y) where X is the element of the first group and Y is the element of the second group. Further with the help of each pair, two triplets of the form (X, Y, W) and (X, Y, W'), where $W = (X + Y + a-1)$ and $W' = (X + Y + a)$, are made. The value of W and W' thus obtained may be greater than $3a$, as these are to lie between $4a + 1$ and $3a$. If these values of W and W' are greater than $3a$, reduce them by a. Thus $2a^2$ triplets are finally obtained such that every symbol appears $2a$ times, any two symbols belonging to two different groups appear 2 times and any two symbols belonging to the same group appear 3 times. Let these $2a^2$ triplets be numbered from 1 to $2a^2$ in any arbitrary manner, Treating $3a$ symbols as treatments and $2a^2$ triplets as blocks, a semi-regular group divisible design, say D^*, with the following parameters is obtained :

$v^* = 3a,\ b^* = 2a^2,\ r^* = 2a,\ k^* = 3,\ \lambda_1^* = 0,\ \lambda_2^* = 2,\ m = 3$ and $n = a$.

The pseudo variance-covariance matrix (Ω^*) of this design can be obtained as follows :

$$\Omega^*_{3a \times 3a} \begin{bmatrix} \Omega_1^* & 0 & 0 \\ 0 & \Omega_1^* & 0 \\ 0 & 0 & \Omega_1^* \end{bmatrix}$$

where $\Omega_1^* = (1/4a^2)\ (3aI_a - E_{a\times a})$, 0 is a null matrix of order $a \times a$, $E_{a\times a}$ is $a \times a$ matrix having each element unity and I_a is a unit matrix of order a.

The dual of this design D^* will be a PBIB (3) design with the following parameters:

$v = 2a^2, b = 3a, r = 3, k = 2a, \lambda_1 = 2, \lambda_2 = 1, \lambda_3 = 0, n_1 = 3,$

$n_2 = 6a - 9$ and $n_3 = 2a^2 - 6a + 5$.

We shall call this design as D.

Alternatively, the design D can be obtained in a compact manner by developing the following three initial blocks :

A : 1, 2, 3, 4, . . . 2a

B : 0+1, 0+2, 2a+1, 2a+2, 4a+2, 4a+1, . . . 2a (a−1) + 1, 2a (a−1) + 2

C : 0+2, 0+3, 2a+4, 2a+5, 4a+6, 4a+7, . . . 2a (a−1) + 1, 2a (a − 1) + 2

(i) Block A is to be developed by increasing each element by 2a till 'a' blocks are obtained. No reduction of the elements is necessary.

(ii) Each of blocks B and C is to be developed by increasing the second term of each element by 2 (keeping the first term the same) till 'a' blocks are obtained. When second part of an element exceeds 2a in course of development, it is to be reduced by 2a. Finally, the two terms of each element are added to get the treatment numbers in the blocks.

(iii) The first associate of any number Z are $Z + 1$, $Z - 1$ and a number, G to be found. If Z is divisible by 2a then $Z + 1$ is to be replaced by $Z + 1 - 2a$. If 1 is the remainder when Z is divided by 2a, then $Z - 1$ is to be replaced by $Z + 1 - 2a$.

From among the blocks obtained by developing C the block containing Z is taken. This block contains either $Z - 1$ or $Z + 1$. Let M denote the number, that is, that one of the $Z + 1$ and $Z - 1$ which is not present in the block containing Z. Let R be the remainder when M is divided by 2a. Subtract R from each number in the block of Z. One of these differences is divisible exactly by 2a. If this difference is denoted by H, then $G = R + H$.

By little inspection the three blocks each containing Z can be located easily. All the numbers in these blocks excepting Z, $Z + 1$, $Z - 1$ and G are its second associates. The remaining numbers are its third associates.

3 Analysis

As the design D is the dual of D^* which is a semi-regular group divisible design, the P method of analysis will be appropriate. The effect of the jth block from the usual additive model.

$$Y_{ij} = \mu + t_i + b_j + e_{ij}, \quad \begin{array}{l} i = 1, 2, \ldots, 2a^2 \\ j = 1, 2, \ldots, 3a \end{array}$$

will be estimated by

$$\hat{b}_j = (1/4a^2)\, 3a\, P_j - \Delta_{(j)} \tag{1}$$

where P_j is the adjusted block total of the jth block and $\Delta_{(j)}$ is the sum of all P values of the group to which j belongs.

Now $$\text{Block } S \cdot S \text{ (adj)} = \Sigma\, \hat{b}_j P_j \tag{2}$$

Having obtained the estimates of the block effects and block $S \cdot S$ (adj), the estimate of the treatment effects and the treatment $S \cdot S$ (adj) are given by

$$\hat{t}_i = (1/r) \text{ (Total of } i\text{th treatment—the sum of the block effects which contain the } i\text{th treatment)} \tag{3}$$

and

$$\text{Tr. } S \cdot S \text{ (adj)} = \Sigma\, \hat{b}_j P_j + (1/r)\, \Sigma\, T_i^2 - (1/k)\, B^2 \tag{4}$$

respectively, where T_i and B_j are the total of ith treatment and jth block respectively.

4. Variances

The pseudo variance-covariance matrix (Ω) of the design D is related to that of D^* through the relation given by Chaudhary and Singh [1] as follows ;

$$\Omega = (1/3)\, I_v - (1/9)\, N' \Omega^* N \tag{5}$$

where N is the incidence matrix of the design D.

This matrix Ω gives the following three different variances between the adjusted treatments means :

(a) $[(4a+1)/6a]\sigma^2$, for the treatments which are I associates;

(b) $[(4a+2)/6a]\sigma^2$, for treatments which are II associates;

(c) $[(4a+3)/6a]\sigma^2$, for the treatments which are third associates.

The efficiency factor of the design D is

$$\text{E.F.} = [2\,(2a^2-1)/(4a^2+3a-5)],$$

which is quite high for $a > 3$.

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